Hierarchical Type-2 Neuro-Fuzzy Model

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***Abstract*—This paper present the model used for forecast the 11 time series from the NN3 Competition 2007. This model is a new type-2 fuzzy inference system for treatment of uncertainties with learning automatic, denominated Type-2 Hierarchical Neuro-Fuzzy Model (T2-HNF). This new model combines the paradigms of modelling of the type-2 FIS and neural networks (NN) with techniques of recursive BSP partitioning. This model, besides to have the capacity to create and to expand its structure automatically, to reduce the limitation referred to the number of inputs and to extract rules of knowledge from a dataset, is also able to model and to manipulate existing uncertainties in real systems, diminishing the effects of these and presenting, consequently, a better performance. Also, this model provides an interval of confidence for its output, that constitutes an important information for real applications. In this context, this model surpasses the limitations of the type-2 fuzzy inference system (type-2 FIS) and the type-1 fuzzy inference systems (type-1 FIS).**

The developed model was evaluated in diverse databases benchmark and real applications of forecast and approximation of functions. The results obtained were compared with others models, demonstrating that T2-HNF model offers next results and in several cases superior to the best results provided by the models used for comparison. In terms of computational time, its performance also is very good. Also, the obtained intervals of confidence for the defuzzified outputs always show to be coherent and offer greater credibility in most of cases when compared with intervals of confidence obtained by traditional methods.

***Index Terms*—Type-2 fuzzy logic systems, interval type-2 fuzzy sets, lower and upper membership functions, hierarchical neuro-fuzzy models, uncertainties.**

I. Introduction

THE concept of information is intimately related to the one of uncertainties. Uncertainties are the result of deficient and not reliable information, of the randomness in the data and the process that generate them (random processes), or of the modelling of variant systems in the time of not known form. Uncertainty is a inherent part to fuzzy inference systems (FIS) used in real applications. The following sources of uncertainties can be present in FIS [1]:

- uncertainties in relation to the meaning of the words used in the antecedents and consequents of the linguistic rules (fuzziness);
- uncertainties in relation to the consequent of one rule (strife);
- uncertainties in relation to the noisy data that activate the FIS and that are used to tune the parameters of this FIS (not specificity);
- uncertainties in relation to the specification of the membership functions of each variable (example: center, end-points).

The type-1 FIS use type-1 fuzzy sets characterized by two dimensions and precise membership functions, where the membership grade is a number in the interval [0,1]. This way, they have limited capacity to model uncertainties totally and directly [1],[2],[3].

The type-2 FIS use type-2 fuzzy sets characterized by membership functions fuzzy, where the membership grade for each element of this set is a fuzzy set in [0.1]. This way, the type-2 membership functions have three dimensions; this new third dimension provides an additional degree of freedom. Besides, these membership functions include a footprint of uncertainty doing possible the quantification and the direct modelling of uncertainties.

The type-2 FIS provide a fundamental measure of dispersion, similar to the variance, to capture more information about its uncertainties in the design of a model. This measure of dispersion constitutes the new direction for FIS. Also, the output set of a type-2 FIS, called type-reduced set, is truely unique and provides an interval of confidence for its output, contributing, this way, more information that an precise output [1],[2],[3].

The type-2 FIS have demonstrated better performance in specific applications in which uncertainty exists, or in applications that have greater complexity, such as non-linearity, non-stationarity or time-variability.

Unfortunately many of the elaborated type-2 FIS present limitations with regard to the reduced number of inputs allowed and to the limited (or nonexistent) form to create their own structure and rules. On the other hand the type-1 FIS have limited capacity to model uncertainties completely, and they do not provide intervals of confidence for their outputs.

In function of the limitations of the type-1 FIS and the type-2 FIS existing, the main objective of this work is to create and to develop a hybrid model type-2 neuro-fuzzy that besides to have the capacity to create and to expand his structure automatically, to reduce the limitation referred to the number of inputs and to extract rules of knowledge from a dataset, is also able to model and to manipulate existing uncertainties in real systems, diminishing the effects of these and presenting, consequently, a better performance. In

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addition, this model must provide an interval of confidence for its output, that constitutes a information more rich than the contained in precise output. Of this form the new T2-HNF model is defined.

The remainder of this paper this organized of the following form. Section 2 introduces T2-HNF model, describing his basic cell, its architecture and his method of learning. Finally, section 3 presents the conclusions.

II. Type-2 Hierarchical Neuro-Fuzzy Model

This model is made up of basic cells, calls type-2 neuro-fuzzy cell. These cells are arranged in a hierarchical structure in the form of a binary tree, in which the cell in the highest level of the hierarchy generates the system's output, while the cells in the lowest level of the hierarchy behave as consequents of the higher hierarchy cells [4].

In this new hybrid model, the type-2 fuzzy rules are generated by an automatic process of partitioning of the input space, using an extension of BSP partitioning for type-2 fuzzy regions. The hierarchical aspect is related to the fact that each partition of the input space defines a type-2 subsystem, that, as well, may have a type-2 subsystem with the same structure as its consequent (recursiveness).

A. Basic Type-2 Neuro-Fuzzy Cell

An type-2 neuro-fuzzy cell [4]is a type-2 neuro-fuzzy mini system that performs binary fuzzy partitioning of to certain space, through its input variable x, according to the type-2 sigmoid membership functions, $\tilde{\rho}$ (low) and $\tilde{\mu}$ (high), described in Fig. 1. In this figure, $\overline{\mu}_{\tilde{\rho}}(x)=\overline{\rho}(x)$, $\underline{\mu}_{\tilde{\rho}}(x)=\underline{\rho}(x)$, $\overline{\mu}_{\tilde{\mu}}(x)=\overline{\mu}(x)$, $\underline{\mu}_{\tilde{\mu}}(x)=\underline{\mu}(x)$ are the upper and lower membership grades of the interval type-2 fuzzy sets low $\tilde{\rho}$ and high $\tilde{\mu}$, respectively.

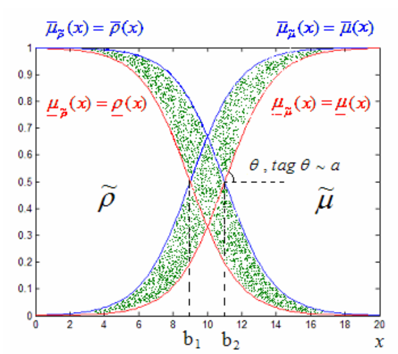


Fig. 1. Profiles of the type-2 membership functions of the T2-NF cell.

The primary membership functions of the sigmoid type-2 membership functions - $\tilde{\rho}$ e $\tilde{\mu}$ - are given by (1) e (2):

$$\mu_{\rho}(x)=sig[a,b,x]=\frac{1}{1+e^{-a(x-b)}}, \quad b\in[b_1,b_2] \tag{1}$$

$$\mu_{\mu}(x)=1-\mu_{\rho}(x) \tag{2}$$

In T2-NF cell, the type-2 membership functions $\tilde{\rho}$ and $\tilde{\mu}$ are implemented so that:

$$\overline{\rho}(x)+\underline{\mu}(x)=1\ ; \quad \underline{\rho}(x)+\overline{\mu}(x)=1\,. \tag{3}$$

Equation (1) describes the sigmoid primary membership function of $\tilde{\rho}$ and $\tilde{\mu}$ with inclination 'a', of fixed value, and middle point of the transition with uncertainty that assumes values in [b_1, b_2]. In T2-NF cell, $\underline{\rho}(x)$ and $\overline{\mu}(x)$ have like middle point of the transition 'b_1'; on the other hand $\overline{\rho}(x)$ and $\underline{\mu}(x)$ have like middle point of transition 'b_2', in agreement shown in the Fig 1.

The linguistic interpretation of the mapping implemented by the T2-NF cell is given by the following set of rules:

rule 1: IF $x\in\tilde{\rho}$ then $y=\subset_1\ ;\subset_1=[c_{1_1},c_{2_1}]$.
(partition 1)
or
rule 2: IF $x\in\tilde{\mu}$ then $y=\subset_2\ ;\subset_2=[c_{1_2},c_{2_2}]$.
(partition 2)

Each rule corresponds to one of the two partitions generated by BSP partitioning. Each partition can in turn be subdivided into two parts by means of another T2-NF cell.

This way, T2-NF cell represents rules whose antecedents are defined by both sigmoid interval type-2 fuzzy sets associated to the input variable x. The value of the input variable is inferred in the sigmoid interval type-2 fuzzy sets of the antecedents. If the antecedent is true, the rule is shot. The consequents, symbolized by $\subset_i$, are interval type-1 fuzzy sets and to be represented by their left and right end-points c_{1_i}, c_{2_i}, respectively, where i indicates *i-th* rule of the system.

Therefore, the type-2 rules in this model let us simultaneously account for uncertainty about antecedent membership functions and consequent parameter values.

Each consequent $\subset_i=[c_{1_i},c_{2_i}]$ corresponds to one of the three possible consequents: a interval type-1 fuzzy set; a linear combination of inputs with interval type-1 fuzzy sets; The output of a cell of a previous level.

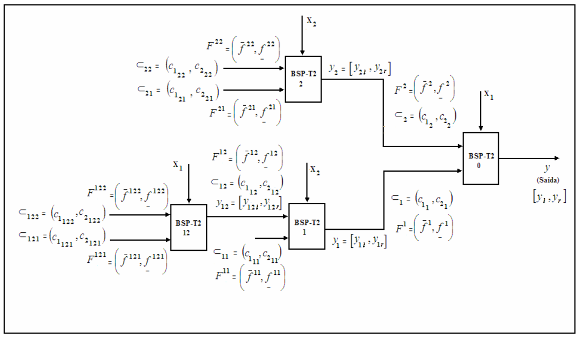
T2-NF cell generates an output that is a interval type-1 fuzzy sets, represented by its left and right end-points y_l, y_r, respectively.

B. T2-HNF Architecture

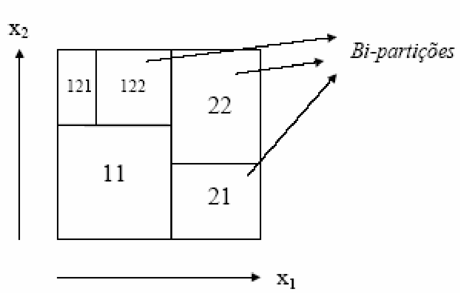
T2-HNF model can be created based on the interconnection of several T2-NF cells. These cells are arranged in a hierarchical structure in the form of a tree; only in the root cell is made a defuzzification.

Fig. 2 illustrates an small T2-HNF architecture along with the respective partitioning of the input space. In this architecture, the initial partitions 1 and 2 ('BSP-T2 0' cell) have been subdivided; hence, the consequents of its type-2 rules are the outputs (interval type-1 fuzzy sets) of subsystem 1 and subsystem 2, i.e., $y_1=[y_{1l},y_{1r}]$ and $y_2=[y_{2l},y_{2r}]$, respectively. In turn, subsystem 1 has as consequents

$\subset_{11} = [c_{1_{11}}, c_{2_{11}}]$ and $y_{12} = [y_{12l}, y_{12r}]$, while subsystem 2 has $\subset_{21} = [c_{1_{21}}, c_{2_{21}}]$ and $\subset_{22} = [c_{1_{22}}, c_{2_{22}}]$ as its consequent. The interval type-1 fuzzy set, consequent $y_{12} = [y_{12l}, y_{12r}]$, is the output of the 'BSP-T2 12' cell.



(a)



(b)

Fig. 2. (a) Example of T2-HNF architecture. (b) BSP Partitioning of the T2-HNF in fig. 2(a).

Each $\subset_i = [c_{1_i}, c_{2_i}]$ corresponds to a interval type-1 fuzzy sets, in the case of using consequents of Sugeno of order 0, or corresponds to one linear combination of the inputs with interval type-1 fuzzy sets, in the case of using consequents of Sugeno of order 1.

C. Learning Algorithm

In the neuro-fuzzy literature the learning process is generally divided in two parts: the identification of the structure and the adjustments of parameters. The T2-HNF model follows the same process. However, only one algorithm carries out both learning tasks simultaneously. The T2-HNF model has a training algorithm based on the gradient descent method [4] for learning the structure of the model (the linguistic type-2 rules) and its fuzzy weights. The parameters that define the profiles of the type-2 membership functions of the antecedents and the left and right end-points of the consequents are regarded as the fuzzy weights of the T2-HNF model. Thus, the $\subset_i = [c_{1_i}, c_{2_i}]$ and the parameters a, b_1 and b_2 are the fuzzy weights of the model. In order to prevent the model's structure from growing indefinitely, a parameter, named decomposition rate ξ ; was used. It is a dimensionless parameter and acts as a limiting factor for the decomposition process.

III. Conclusions

The study of cases, made with different "benchmark" databases and real applications in different areas, confirm the good applicability of this model in the task of forecast and approximation of functions.

The T2-HNF model present better performance in the task of forecast and approximation of functions in cases where it exists greater complexity and greater uncertainties.

The performance of this model in relation to the computational time of processing also was very good, presenting a smaller computational cost in comparison with the other models like MLP NN.

The T2-HNF model has the advantage to model uncertainties in form totally new, being able to give an interval of confidence - important in real applications - for their outputs, contrary to others models that have limitations in the modelling of present uncertainties and the not to give an interval of confidence for their outputs; since these models only provide precise outputs.

The intervals of confidence obtained of automatic form by the T2-HNF model are coherent when they are compared with intervals of confidence obtained by traditional methods. Also is observed that these intervals offer greater credibility in majority of cases, once they are more narrow than the traditional intervals of confidence.

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